B. Math. I Year 2001-2002
I Semester Final Exam
Probability - I

Date: 23-11-2001 **Time: 3 Hours** **Max. Marks: 100**

Note: This paper carries questions for 115 marks. You may answer any part of any question. The maximum you can score is limited to 100.

1. An ordered random sample of size n is drawn from a population of K distinct elements with replacement. Let X be the number of distinct elements in the sample. Find the expectation and the variance of X. [20]

2. Let N be a fixed positive integer. Let X denote the number of failures preceding the Nth success in a sequence of Bernoulli trials with constant probability P of success in each trial. Find the generating function for the distribution of X. Find $E(X)$ and $V(X)$ using the generating function or otherwise. [20]

3. Let X be a discrete random variable taking nonnegative integer values and with finite expectation. Show that $E(X) = \sum_1^\infty r_n$ where $r_n = P(X \geq n)$. Hence find $E(\min(X_1, X_2, \ldots, X_k))$ in terms of r_n where $X_1, X_2, \ldots, X_k$ are independent and identically distributed as X. [15]

4. (a) Prove Chebychef's inequality for a discrete random variable finite variance.

   (b) State and prove one form of the weak law of large numbers.

   [10 + 10]

5. Let X be a geometric random variable with distribution given by $P(X = n) = (1 - \alpha)^{n-1}\alpha$ for all $n \geq 1$. Given that $X = n$, let Y be a Binomial random variable with parameters n and p. Find the unconditional distribution of Y. [15]